Uncertainty Analysis of Dynamical Systems – Applications to Volcanic Ash Transport

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UQ/UA for Dynamic Systems

• Tough topic/hot topic – 4 sessions MS 54, MS 61, MS 68, MS 76
• Rich diversity of topics
Volcanic Hazard

• Dense fast moving ($O(10) \text{ m/s}$) ground-hugging flows comprised of heterogeneous particulates and water – travel $O(10)$ km

[Stefanescu Thu 2:00] [MS37 Bayarri, Pitman, Spiller, Wolpert]

• Ash transport into atmosphere – wind borne fine particulates travel $O(1000)$ km in atmosphere

[Singla – next session]
The Ash Problem

**Problem:** Currently used forecasts of ash transport in eruption of Eyjafjallajökull, Iceland caused total shutdown of large swathes of airspace, the cancellation of more than 100,000 flights and total disruption!

Significant discrepancy between no-fly zones, actual ash observation, and multiple model forecasts!

**Solution:** Provide *probabilistic map* that can be updated dynamically with observations using a data driven (DDDAS) approach

**Challenges:** Uncertainty Analysis; High fidelity models representing the complex physics capable of needed near real time execution; Data and Workflow Management; Sensor error; measurement mismatch; imagery analysis
Problems

• Physics models are hard (non-linear hyperbolic systems) and simulators are expensive.
  – Non-linear Constitutive laws
  – Model/Numerical Error in Solutions can be large
• Model parameters and forcing terms are uncertain
• Quantity of interest (QOI) – “hazard of flow > threshold over a jurisdiction” is high dimension
• For Ash QOI is also time dependent
• Simulations take $O(1hr)$ – ensembles for UQ are small!

$^1$Savage-Hutter’89, Iverson et. al. ’97,’01,’04,’12, Pitman-Le’05, ...
Approaches

• Simulators using “best-in-practice” numerical methods and modeling
  – TITAN2D\textsuperscript{1} – depth averaged, Mohr-Coulomb granular flow model, parallel adaptive Godunov solver, inputs of initial flow volume, frictional resistance, start location/direction, \textbf{topography}
  – \textit{bent-puff}\textsuperscript{2} -- novel coupled eruption column model + Lagrangian advection/diffusion model of ash transport, inputs of vent parameters, grain size distribution and wind field

\textsuperscript{1}Patra et al ’05, \textsuperscript{2}Bursik et. al in review
Approaches

• For ground flows quantity of interest simplified by looking at “maximum flow depth during each possible flow”

• Input data modeled by suitable distributions can be sampled and outcomes of simulations combined to estimate hazard

• For expensive simulators surrogate based methods (polynomial chaos, Bayes linear models, GaSP) can reduce the overall cost of computation
Probabilistic Hazard Map – for hazardous mass flows at Mammoth Mountain, CA

Ensembles (O(10^2)) of flow simulations using complex and coupled models
Experimental Design over input Data and Model Parameter Ranges (flow volume, terrain error, basal flow resistance, start location)

Surrogate models (Bayes Linear Models, Polynomial Chaos Quadrature)

Construction of Bayes Linear Models parallelized by localizing covariance

Sampling of surrogate for Hazard analysis

Probability of flow depth >1m at different location in the entire jurisdiction.

Computed in ~6hrs on 2TF of computing!

4/4/12
Patra/SUNY at Buffalo/SIAM-UQ
Highlights of Methodology

• Large Jurisdiction Hazard Analysis
  – Effective use of HPC
  – Bayesian Methods are sequential

• Sparsification of flow data
  – Downsample to preserve flow features with less data

• Localization of covariance structure
  – Tesselate input data and physical space
  – Localize by considering only data in “n-hops”
  – Current work: replace tessellation by dynamically constructed tree structure
  – Reconstruct global emulator using barycentric weight scheme

\[ R^{-1} \approx \sum_{i=1}^{M} R_i^{-1} \]

\(^{1}\)Dalbey ‘09, Dalbey et al in review
New Methods to Mitigate the Risk to Aviation

What if it was possible to:

• run stochastic models in real-time to generate a practical assessment of ash risk over large jurisdictions?
• Give estimates of cloud position, height, mass loading and grain size?

Challenges

• Computability (estimates for creating models from data/sim. \(\sim O(100)\) days)
• Adaptability – DDDAS methods

Basic Approach

• **Physics based Coupled Models + Data**
  e.g. BENT+ PUFF + Observations
  \(\rightarrow\) reduced (surrogate) model (polynomial chaos)
  \(\rightarrow\) updated reduced models
  \(\rightarrow\) Probabilistic hazard analysis

Ash retrieval 20 April 2010

– Loading, height, grain size
DDDAS Approach To Volcanic Ash Transport And Dispersal Forecast

CALIPSO/SEVIRI

Satellite Image

Bayes

pdf of Ash Plume

pdf Satellite Ash loading/footprint

PCQ: Ensemble BENT-PUFF

AGMM

High Fidelity Simulator

Source Parameter pdf

Uncertain Wind-Field (Data + NWP)

Source Parameter ID

BENT: Eruption Plume Model
PUFF: Ash transport and Dispersal Model
PCQ: Polynomial Chaos Quadrature
AGMM: Adaptive Gaussian Mixtures
CALIPSO/SEVIRI: Satellite based sensors for ash detection

4/4/12

Patra/SUNY at Buffalo/SIAM-UQ
Approach

\[
\frac{dx}{dt} = f(t_k, x_k, \Theta_k) \quad x(t_0) = x_0
\]

- Study the evolution of pdf \( p(x_k, t_k) \) through time where
  - \( f = \text{bent-puff} \),
  - \( x_k \) state variables (e.g. ash concentrations),
  - time \( t_k \),
  - \( \Theta_k \) parameters

- Use Monte Carlo 🧐

- Replace by Polynomial Chaos Quadrature (PCQ) or ???
Approach

\[ \theta_i(\xi) = \sum_{k=1}^{N} \theta_{ik} \phi_k \]

\[ x_i(t, \Theta) = \sum_{k=0}^{N} x_{ik}(t) \phi_k(\xi) \]

- Galerkin collocation can be used to obtain the weights
- Interchanging order of time and probability integration

\[ \langle x(t)^N \rangle = \sum_q w_q \left( \int_0^t f(t, x, \Theta_q) dt \right)^N \]
Table 1: Eruption source parameters based on observations of Eyjafjallajökull volcano and information from other similar eruptions of the past.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value range</th>
<th>PDF</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vent radius, $b_0$, m</td>
<td>65-150</td>
<td>Uniform, + definite</td>
<td>Measured from IMO radar image of summit vents on 14 April 2010</td>
</tr>
<tr>
<td>Vent velocity, $w_0$, m/s</td>
<td>Range: 45-124</td>
<td>Uniform, + definite</td>
<td>Measured by infrasound (Ripepe et al., 2010) 6-21 May, when MER similar to 14-18 April</td>
</tr>
<tr>
<td>Mean grain size, $M_{d_{φ}}$, $φ$ units</td>
<td>2 boxcars*: 1.5-2 and 3-5</td>
<td>Sum of two uniform, $∈ \mathbb{R}$</td>
<td>(Woods and Bursik, 1991), Table 1, vulcanian and phreatoplinian. A. Hoskuldsson, Eyjafjallajökull Eruption Workshop, 09/2010, presentation, quote: ‘vulcanian with unusual production of fine ash’.</td>
</tr>
<tr>
<td>$σ_φ$, $φ$ units</td>
<td>2.0 ± 0.6</td>
<td>Uniform, $∈ \mathbb{R}$</td>
<td>(Woods and Bursik, 1991), Table 1, vulcanian and phreatoplinian</td>
</tr>
</tbody>
</table>

boxcar: function that is zero everywhere except over a short interval where it is constant
Preliminary Results

Developed parallelized PCQ/Bent-Puff HPC based tool for probabilistic ash forecasting

Physics based methodology for VATD “transport and dispersion” model inputs – poorly characterized column height, mass eruption rate replaced by pdf of observable vent parameters and speed.

PCQ based probabilistic hazard analysis replaces predictions of existing tools.

Results for Eyjafjallojokull are very promising – all ash observed was inside a Probability>0.2 contour with most in Probability >0.7

Only risk based (probabilistic) forecast for ash cloud with full transport modeling!!