Adaptive Magnetohydrodynamics Simulations with SAMRAI

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Outline

• Introduction to AMR and SAMRAI
• Introduction to pixie3d
  • Example problem
  • Demonstration of correctness
  • Scaling
• Introduction to Radiation-Diffusion
  • Sample results
  • Scaling
• Challenges / Conclusions
Structured Adaptive Mesh Refinement

Structured adaptive mesh refinement (SAMR) represents a locally refined mesh as a union of logically rectangular meshes.
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SAMRAI

Structured Adaptive Mesh Refinement Application Infrastructure (SAMRAI)

• Application for structured AMR meshes
• Refinement based on cell tagging
• Flexible refinement ratio and box sizes
• Each patch can exist on different processors
  • Allows for easy parallelization
• Each processor writes a separate file every timestep
  • Simple parallel IO
  • HDF5 based file format
• v3.3.1
  • Recent redesign to allow for better scaling
  • Removed knowledge of all boxes from each processor
  • Interfaces changed
94,525 cells in simulation
3,360,000 cells needed to cover domain at fine resolution
Only 2.8% of the cells are needed
36x improvement
As the plasma evolves, the tagging strategy checks the gradients and determines if additional resolution is necessary.
Extended MHD Model Equations

<table>
<thead>
<tr>
<th>Fluid Eqs.</th>
<th>Maxwells Eqs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 ]</td>
<td>[ \partial_i \vec{B} + \nabla \times \vec{E} = 0 ]</td>
</tr>
<tr>
<td>[ \frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v} + \vec{\Pi}) = \vec{j} \times \vec{B} - \nabla p ]</td>
<td>[ -\mu_0 \varepsilon_0 \partial_i \vec{E} + \nabla \times \vec{B} = 0 ]</td>
</tr>
<tr>
<td>[ \frac{\partial p}{\partial t} + \sum_{s=i,e} \left[ \vec{v}_s \cdot \nabla p_s + \gamma p_s \nabla \cdot \vec{v}_s \right] = (\gamma - 1)(Q - \nabla \cdot \vec{q}) ]</td>
<td>[ \nabla \cdot \vec{B} = 0 ]</td>
</tr>
<tr>
<td></td>
<td>[ \nabla \cdot \vec{E} = \frac{e(n_i - n_e)}{\varepsilon_0} ]</td>
</tr>
</tbody>
</table>

On scales of interest (both temporal and spatial), plasmas are quasineutral:

\[ n_i \approx n_e \quad \mu_0 \vec{j} = \nabla \times \vec{B} \]

However, \( \nabla \cdot \vec{E} \neq 0 \) Electric field is determined from electron equation of motion:

\[ \text{Ohm’s Law:} \quad \vec{E} \approx -\vec{v} \times \vec{B} + \eta \vec{j} + \frac{d_i}{\rho} \left( \vec{j} \times \vec{B} - \nabla p_e - \nabla \cdot \vec{\Pi} \right) \]

Electric field equation determines the nature of MHD model (ideal, resistive, extended)
Resistive MHD Equations

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0
\]

\[
\frac{\partial \vec{B}}{\partial t} + \nabla \times \vec{E} = 0
\]

\[
\frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot \left[ \rho \vec{v} \vec{v} - \vec{B} \vec{B} + \vec{\Pi} + \vec{I} \left( p + \frac{B^2}{2} \right) \right] = 0
\]

\[
\frac{\partial T_e}{\partial t} + \vec{v} \cdot \nabla T_e + (\gamma - 1) T_e \nabla \cdot \vec{v} = (\gamma - 1) \left( \frac{Q - \nabla \cdot \vec{q}}{(1 + \alpha) \rho} \right)
\]

\[
\alpha = \frac{T_e}{T_i} \quad \vec{\Pi} = -\rho \vec{v} \nabla \vec{v} \quad \vec{q} = -k \nabla T = -k(1 + \alpha) \nabla T_e
\]

\[
\vec{E} = -\vec{v} \times \vec{B} + \eta \vec{j}
\]
pixie3d Results

64x64x1, 4 procs

128x128x1, 16 procs

256x256x1, 64 procs
# Island Coalescence Scaling

<table>
<thead>
<tr>
<th></th>
<th># procs</th>
<th># iterations</th>
<th>Time (seconds)</th>
<th>Time / iteration</th>
<th>% Comm</th>
</tr>
</thead>
<tbody>
<tr>
<td>64x64 1 level</td>
<td>4</td>
<td>4099</td>
<td>1006</td>
<td>0.25</td>
<td>33%</td>
</tr>
<tr>
<td>128x128 1 level</td>
<td>16</td>
<td>10500</td>
<td>2453</td>
<td>0.23</td>
<td>20%</td>
</tr>
<tr>
<td>256x256 1 level</td>
<td>64</td>
<td>21672</td>
<td>6685</td>
<td>0.31</td>
<td>27%</td>
</tr>
<tr>
<td>128x128 2 level *</td>
<td>432</td>
<td>21651</td>
<td>7661</td>
<td>0.35</td>
<td>56%</td>
</tr>
</tbody>
</table>

* The 2 level run was not made for the scaling study and is over decomposed
pixie3d Results

DB: pixie3d.h5
Cycle: 0
Time: 0

X-Axis

Y-Axis

user: Mark Berrill
Mon Feb 13 15:40:42 2012
\( \text{Rho} \)  \( \text{T}_e \)  \( \text{V}_x \)  \( \text{V}_y \)
pixie3d Results

DB: summary.samrai
Cycle: 0
Time: 0

Pseudocolor
Var: J over z
-0.8291
-0.4146
-2.011e-011
-0.4146
-0.8291

Max: 0.8291
Min: -0.8291

X-Axis

Y-Axis

user: Mark Berrill
Tue Feb 14 20:07:16 2012
Non-Equilibrium Radiation-Diffusion

Model Equations

\[
\frac{\partial E}{\partial t} - \nabla \cdot (D_r \nabla E) = \sigma_a \left(T^4 - E\right) \quad \text{in} \quad \Omega = [0,1]^d
\]

\[
\frac{\partial T}{\partial t} - \nabla \cdot (D_t \nabla T) = -\sigma_a \left(T^4 - E\right) \quad \text{in} \quad \Omega = [0,1]^d
\]

Constitutive law:

\[
\sigma_a = \frac{Z^3}{T^3}
\]

Diffusion coefficients:

\[
D_r = \frac{1}{3\sigma_a + \frac{\|\nabla E\|}{E}}
\]

\[
D_t = kT^{5/2}
\]
Non-Equilibrium Radiation-Diffusion

Model Equations

\[ \frac{\partial E}{\partial t} - \nabla \cdot (D_r \nabla E) = \sigma_a (T^4 - E) \quad \text{in} \quad \Omega = [0,1]^d \]

\[ \frac{\partial T}{\partial t} - \nabla \cdot (D_t \nabla T) = -\sigma_a (T^4 - E) \quad \text{in} \quad \Omega = [0,1]^d \]

Initial conditions:

\[ E = E_0 \quad T = E_0^{1/4} \quad \text{at} \quad t = 0 \]

Boundary conditions:

\[ \frac{1}{2} n \cdot D_r \nabla E + \frac{E}{4} = R \quad \text{on} \quad \partial \Omega_R, \ t \geq 0 \]

\[ n \cdot D_r \nabla E = 0 \quad \text{on} \quad \partial \Omega_N, \ t \geq 0 \]

\[ n \cdot \nabla T = 0 \quad \text{on} \quad \partial \Omega, \ t \geq 0 \]
Non-Equilibrium Radiation-Diffusion
# Non-Equilibrium Radiation-Diffusion

<table>
<thead>
<tr>
<th></th>
<th># proc</th>
<th># Time Steps</th>
<th>Time (minutes)</th>
<th>Time / step</th>
<th>% of time in refine</th>
</tr>
</thead>
<tbody>
<tr>
<td>16x16x1 1 level</td>
<td>1</td>
<td>2044</td>
<td>5.2</td>
<td>0.15</td>
<td>4%</td>
</tr>
<tr>
<td>16x16x16 2 level</td>
<td>8</td>
<td>2787</td>
<td>28.4</td>
<td>0.61</td>
<td>41%</td>
</tr>
<tr>
<td>16x16x16 3 level</td>
<td>64</td>
<td>2702</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16x16x16 4 level</td>
<td>512</td>
<td>6593</td>
<td>642</td>
<td>5.84</td>
<td>47%</td>
</tr>
<tr>
<td>32x32x32 1 level</td>
<td>8</td>
<td>3068</td>
<td>23.4</td>
<td>0.46</td>
<td>19%</td>
</tr>
<tr>
<td>32x32x32 2 level</td>
<td>64</td>
<td>5189</td>
<td>126</td>
<td>1.46</td>
<td>44%</td>
</tr>
<tr>
<td>32x32x32 3 level</td>
<td>512</td>
<td>7106</td>
<td>528</td>
<td>4.46</td>
<td>48%</td>
</tr>
<tr>
<td>64x64x64 1 level</td>
<td>64</td>
<td>4305</td>
<td>114</td>
<td>1.59</td>
<td>43%</td>
</tr>
<tr>
<td>64x64x64 2 level</td>
<td>512</td>
<td>8004</td>
<td>450</td>
<td>3.37</td>
<td>42%</td>
</tr>
</tbody>
</table>
Conclusions / Future Work

• Basic application is working
• Implicit methods are necessary
  • Preconditioning likely needed
• Adaptive meshing is in place
  • Need to test cell tagging
• Need to do a better job at coarse-fine interfaces
  • Preserve conservation
  • Preserve divergence of $B = 0$
  • Higher order interpolation likely necessary
• Parallel performance needs more attention
  • Current runs are promising
  • Load balancer creates an excessive number of patches

* See Scalable Physics-based Preconditioning for 3D
  Extended MHD in MS55 for more on pixie3d