Inverse Problems in Power System Dynamics

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Motivation

• Analysis of real-world systems is challenging.
• Investigations rely on simulation.
  – Typically address forward problems.
  – Can more information be obtained?
• Many analysis questions take the form of inverse problems.

System identification  Causation
Hybrid dynamical systems

Characterized by

- Continuous and discrete states
- Continuous dynamics
- Discrete events (triggers)
- Mappings that define the evolution of states at events

Compass gait biped robot (mechanical processes)

Power systems (electrical processes)
Hybrid system modelling

**Differential Algebraic Impulsive Switched (DAIS) model**

- Starting point is the familiar DAE model
  \[
  \begin{align*}
  \dot{x} &= f(x, y) \\
  0 &= g(x, y)
  \end{align*}
  \]

- Switched events have the form
  \[
  g(x, y) = \begin{cases} 
  g^-(x, y) & s(x, y) < 0 \\
  g^+(x, y) & s(x, y) > 0 
  \end{cases}
  \]

- Impulsive events
  \[
  x^+ = h(x^-, y^-) \quad \text{when} \quad s(x, y) = 0
  \]

The model generates the (piecewise smooth) *flow*
\[
  x(t) = \phi(x_0, t), \quad y(t) = \psi(x_0, t)
  \]
Trajectory sensitivities

- Linearize the system around a trajectory rather than around the equilibrium point.
  \[ \Delta x(t) = \frac{\partial \phi(x_0, t)}{\partial x_0} \Delta x_0 + \text{higher order terms} \approx \Phi(t) \Delta x_0 \]

- Determine the change in the trajectory due to (small) changes in parameters and/or initial conditions.
  - Parameters incorporated via \( \dot{\lambda} = 0, \quad \lambda(0) = \lambda_0 \)

- Provides gradient information for iteratively solving inverse problems.
Trajectory sensitivity evolution

• Along smooth sections of the trajectory

System evolution

\[ \dot{x} = f(x), \quad x(0) = x_0 \]

Sensitivity evolution

\[ \dot{\Phi} = \left. \frac{\partial f}{\partial x} \right|_{x(t)} \Phi, \quad \Phi(0) = I \]

• At an event

\[ \Phi(\tau^+) = \Phi(\tau^-) - (f^+ - f^-) \frac{\partial \tau}{\partial x_0} \]
Implicit numerical integration allows efficient computation of trajectory sensitivities.

System evolution
\[ \dot{x} = f(x) \]

Trapezoidal integration
\[ x^{k+1} = x^k + \frac{h}{2} \left( f(x^k) + f(x^{k+1}) \right) \]

Each integration timestep involves a Newton solution process.
- The Jacobian \( \left( \frac{h}{2} Df - I \right) \) must be formed and factored.

Sensitivity evolution
\[ \hat{\Phi} = Df(x(t)) \Phi \]

Trapezoidal integration
\[ \Phi^{k+1} = \Phi^k + \frac{h}{2} \left( Df(x^k) \Phi^k + Df(x^{k+1}) \Phi^{k+1} \right) \]
\[ \Rightarrow \left( \frac{h}{2} Df(x^{k+1}) - I \right) \Phi^{k+1} = -\left( \frac{h}{2} Df(x^k) + I \right) \Phi^k \]

Already factored
**Parameter uncertainty**

**Worst-case analysis:** Parameter uncertainty is uniformly distributed over an orthotope $\mathcal{B}$ (multi-dimensional rectangle.)

Assume all trajectories emanating from $x_0 + \mathcal{B}$ have the same order of events.

**Trajectory approximation:**
Neglecting higher order terms of the Taylor series,

$$
\phi(x_0 + \Delta x_0, t) 
\approx \phi(x_0, t) + \Phi(x_0, t)\Delta x_0
$$

Propagation of uncertainty is described (approximately) by the time-varying parallelootope,

$$
\mathcal{P}(t) = \phi(x_0, t) + \Phi(x_0, t)\mathcal{B}
$$
Parameter estimation

- Determine which parameters are *well conditioned*.
- Estimate those parameters.
- Nonlinear least-squares problem.

Real world example:
- Disturbance on the 330kV Scandinavian network.
- A voltage measurement was used to estimate various parameters, including the *switching time* of an important switched reactor.
Dynamic embedded optimization

Problem formulation:

\[ \min_{\theta} \mathcal{J}(x, \theta) \]
subject to \( x(t) = \phi(x_0(\theta), t) \)

Cost function may have the form:

\[ \mathcal{J}(x, \theta) = \mathcal{E}(x(t_f), \theta) + \int_{t_0}^{t_f} \mathcal{L}(x(t), \theta, t) dt \]

Assumption: Order of events does not change as \( \theta \) varies.
- Switching surfaces are always transversally crossed, no grazing.
- All flows have the same history.
- Trajectory sensitivities exist.

If \( \mathcal{J} \) is a smooth function of its arguments, then it is continuously differentiable with respect to \( \theta \).
- Gradient-based algorithms are applicable.
Optimization example

Improve damping by optimizing PSS limits.

Optimization adjusted lower PSS limit from -0.1 to –0.33.
Limit cycles (periodic behaviour)

- Analysis and computation are based on Poincaré map concepts.
- Solve \( F_l(x) := \phi(x, \tau_r(x)) - x = 0 \) where \( \tau_r(x) \) is the return time.
- Reliable convergence, even for unstable limit cycles.

- Limit cycles may be non-smooth.
- **Example:** Interactions between a tap-changing transformer and a switched capacitor.
Grazing phenomena

- Tangential encounter between the trajectory and a specified surface.
- Solve

\[ \phi(x_0(\theta_g), t_g) - x_g = 0 \]
\[ b(x_g) = 0 \]
\[ \nabla b(x_g) \top f(x_g) = 0 \]

Example: Distance protection – determine bounding value of parameters that induce protection operation.
Shooting methods

- **Point solutions**: Solve \( F(z) = 0 \) where \( F \) incorporates the flow \( \phi \).
  - Newton solution: \( z^{k+1} = z^k - DF(z^k)^{-1} F(z^k) \)
  - Evaluation of \( F \) requires simulation to determine \( \phi \).
  - Evaluation of \( DF \) requires trajectory sensitivities \( \Phi \).

- **Continuation process**: Under-determined system \( F : \mathbb{R}^{n+1} \to \mathbb{R}^n \)
  - Solutions of \( F(z) = 0 \) describe a 1-manifold.

- **Optimization**.
Example: bifurcations

Single machine infinite bus system.
Conclusions

• Most real-world systems are hybrid (exhibit interactions between continuous dynamics and discrete events).

• Systematic modelling of hybrid systems enables:
  – Well defined and efficiently computed trajectory sensitivities.
  – Gradient-based techniques for solving optimization and boundary value problems.
  – Algorithms for moving beyond forward problems.